Fiabilité dépendant du temps : quelques considérations nouvelles pour en favoriser l’application

Time-variant reliability: some new considerations for enhancing industrial application

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Résumé
Cet article présente quelques améliorations théoriques à la théorie de la fiabilité des structures dépendant du temps. Il est bien connu que le cas dit « à marge décroissante » (cas où la fonction d’état-limite G est décroissante en fonction du temps) est d’une importance pratique notable : dans ce cas, le calcul de la probabilité de défaillance cumulée se ramène au cas d’une probabilité instantanée, qui relève de méthodes classiques. Cet article confirme que de nombreux mécanismes de dégradation affectant les centrales nucléaires relèvent de ce cas. Ensuite, il montre deux extensions possibles : tout d’abord, une expression de la cinématique de dégradation plus générale, et deuxièmement, le cas de la « défaillance unique » conduisent à la même solution. Enfin, cet article aborde la comparaison entre les taux de défaillance de la fiabilité classique et le taux de franchissement de seuil \( \nu(t) \), et des relations simples entre ces grandeurs sont démontrées. Elles montrent la proximité entre ces deux classes de méthodes fiabilistes. Les taux de défaillance pourraient donc aussi être utilisés en fiabilité des structures.

Summary
This paper presents some theoretical improvements to the background of time-variant structural reliability theory. It is well known that the case of a decreasing G-function (margin) is of great practical importance: in this case the global failure probability reduces to an instantaneous probability, easy to compute. This paper confirms that many degradation mechanisms affecting Nuclear Power Plant facilities correspond to this case. Then, it shows two possible extensions: a more general form of the degradation kinetics as well as the case of “unique failure” lead to a similar situation. Finally, a comparison between failure rates of the classical reliability theory and the outcrossing rate \( \nu(t) \) is performed, and simple relationships are demonstrated. They show the closeness between the two classes of reliability methods. Failure rates could also be used in structural reliability.

1 DESCRIPTION OF THE CONTEXT

Developed societies are getting more and more sensitive to risks, especially due to technological devices and industrial facilities. Safety requirements apply to every kind of industrial or public facilities.

In this context, the reliability of structures is an important safety issue. It is the result of a global and consistent process affecting every step of the structural life cycle: design, manufacturing, installation, operation, maintenance policy, lifetime evaluation… All the measures taken at each of these steps have an impact on the “practical reliability” of the structures.

To give confidence in the reliability level of the structures, or simply to comply with the regulatory requirements in some cases, this reliability level has sometimes to be evaluated.

Consequently, structural reliability analyses have been increasingly applied in many industrial branches in the last decades, as in the French nuclear power generation industry, especially for safety-significant components.

Structural reliability analysis aims at computing the probability of failure of a mechanical system with respect to a prescribed failure criterion by accounting for uncertainties affecting the input variables (geometry, material properties, loading) of the physical model.

In its simplest form, the failure criterion can be expressed as a “stress” term \( S \) exceeding a resistance term \( R \), i.e. \( R \leq S \), or, more generally, \( R(X_1, ..., X_p) \leq S(X_{p+1}, ..., X_n) \), and the general time-invariant structural reliability problem consists in calculating the instantaneous failure probability

\[ P_f = P(R(X_1, ..., X_p) \leq S(X_{p+1}, ..., X_n)) \]

at a given instant \( t \). In very specific cases, common variables may appear in the \( R \) and \( S \) terms.

However, these quantities \( R \) and \( S \) are not constant over time:

- The structural components of industrial facilities can be submitted to aging and deterioration processes: degradation mechanisms (corrosion, irradiation, etc…) may affect the structure, and the material properties decay in time;
- The applied loads resulting in \( S \) may be varying in time (wind velocity, temperature or wave height…): stochastic processes are introduced.
Moreover, in the context of durability analysis and lifetime evaluation of structures (a major concern in the industrial applications of EDF, which owns a numerous fleet of nuclear power plants), it is necessary to evaluate the structural reliability over a given period (e.g. the anticipated lifetime, or an operation cycle), and not only to make an instantaneous evaluation giving a point-in-time failure probability (time-invariant reliability).

Time-variant reliability aims at assessing the probability of failure of a structure over a given lifetime.

In comparison with time-invariant reliability where the reliability is evaluated at a given time, the specificity of time-variant reliability is due to the fact that the failure instant is generally not known a priori for a particular trajectory, and varies from a trajectory to another. The theoretical background is briefly summarized in the sequel.

2 TIME-VARIANT RELIABILITY: BASIC THEORETICAL FRAMEWORK

This description is mainly derived from (Sudret, 2002).

2.1 Problem statement

Denoting by \([0, T]\) the time interval under consideration, the probability of failure of the structure within this time interval will be called "global failure probability" and is defined as follows:

\[
P_f(0,T) = P(\exists \, t \in [0, T] / G(t, X(t, \omega)) \leq 0)\]  \(1\)

This quantity is a priori different from the so-called point-in-time (i.e. instantaneous) probability of failure defined as follows:

\[
P_f(t) = P(G(t, X(t, \omega)) \leq 0)\]  \(2\)

2.2 Case of decreasing performance function \(G\)

When only degradation mechanisms are considered, or when the effect of action is monotonously increasing (cf. concrete swelling due to the alkali-aggregate reaction), the limit state function \(G\) is usually monotonously decreasing in time, meaning that all trajectories of \(G\) are decreasing. If this property can be established, it can be seen easily that \(P_f(0,T)\) reduces to \(P_f(T)\): it is a right-boundary problem. This case is of great practical importance, as will be confirmed in this chapter.

2.3 General case: outcrossing theory

When random loading is considered, no such simplified result can be used. The usual approach is then the outcrossing approach. Denoting by \(N(0, T)\) the number of outcrossings through the limit state surface, one can write:

\[
P_f(0,T) = P(G(0, X(0, \omega)) \leq 0) \cup \{ N(0, T) > 0\}\]  \(3\)

In this general case only bounds for the probability are available:

\[
\max_{0 \leq t \leq T} \{P_f(t)\} \leq P_f(0,T) \leq P_f(0) + E[N(0, T)]\]  \(4\)

For safety reasons the upper bound is of particular interest in the industrial applications, whereas the lower bound would lead to underestimate the failure probability, which is not acceptable.

If the processes under consideration are regular, the outcrossing rate \(\nu^+\) is defined as the following limit:

\[
\nu^+(t) = \lim_{\Delta t \to 0} \frac{P(N(t, t+\Delta t) = 1)}{\Delta t}\]  \(5\)

And the mean number of outcrossings reads:

\[
E[N(0, T)] = \int_0^T \nu^+(t) \, dt\]  \(6\)

The key issue and the main difficulty of this theory are therefore the evaluation of \(\nu^+(t)\). At this aim various methods are available and have generated significant research effort: general analytical formulations, and two main classes of numerical methods:

- asymptotic integration (Rackwitz, 1998);
- system reliability approach (the so-called PHg2 method presented for example in (Sudret, 2002)).

In case of stationary processes, the outcrossing rate \(\nu^+\) is constant, and the latter integral simplifies into:

\[
E[N(0, T)] = \nu^+.T\]  \(7\)

In case the \(G\)-function only depends on stationary processes and does not contain random variables, the outcrossing number is a Poisson process (intensity = \(\nu^+\)), and \(P_f(0,T)\) is approximately equal to \(1 - e^{-\nu^+.T}\).

As an example of analytical formulation of the outcrossing rate \(\nu^+\), the most famous and general one is the classical Rice’s formula (Rice, 1945) that reads:

\[
\nu^+(t) = \int_{z(t)}^{\infty} (s - \Delta t(s)) \, f_X(x(t), s) \, ds\]  \(8\)

Where \(z(t)\) : time-variant threshold that can be crossed.
s(t) : random scalar process for which the threshold upcrossing means failure
\( \dot{x}(t) \) : time derivative of the process or function \( x(t) \)

For vector processes this formula can be generalized by Belayev's formula (Belayev, 1968).

The Rice formula provides closed form solutions for Gaussian stationary \( s(t) \) processes and constant or time varying thresholds \( z(t) \) (Sudret, 2002). Its application to Gaussian non stationary processes is possible but requires a temporal integration, since the parameters of the Gaussian process are time varying.

The Rice formula forms the basis of the abovementioned asymptotic integration method. This method estimates \( \nu^*(t) \) and its integral through the Laplace integration method, including various approximations. It has been implemented in the STRUREL/COMREL-TV Software.

The abovementioned PHd2 method (Audrieu, 2004) makes it possible to use the classical time-invariant reliability evaluation methods FORM/SORM, but, consequently, is submitted to the validity of the FORM approximation and needs to optimize the time discretization increment. It can therefore be used (with some appropriate development) in any classical reliability or uncertainty treatment software, like OpenTURNS or Phimecasoft.

These two competing numerical methods are both subjected to approximations, and their comparative evaluations may lead to opposite conclusions (Sudret, 2002, Sykora, 2005).

Besides Gaussian processes, another class of stochastic processes is commonly used in the time-variant reliability literature: the renewal (jump) processes. These processes are supposed to be relevant for modeling the applied load process (e.g. traffic loads). For rectangular renewal processes, the load intensity is assumed to be constant, but other assumptions may be preferable (triangular, parabolic profile, Sykora, 2005). Regular or intermittent processes can be considered. Based on (4), new bounds have been developed for the renewal processes (Sykora, 2005). Note also that for scalar renewal processes, an analytical expression for \( \nu^*(t) \) is available.

### 3 SOME THEORETICAL ENHANCEMENTS

This chapter presents some theoretical improvements to the fundamental background aforementioned.

#### 3.1 Extension of the decreasing limit state function case

As mentioned in section 2, this case is of great practical importance, since the global failure probability reduces to a point-in-time probability that can be evaluated by usual well-established time-invariant reliability methods. It is interesting to investigate whether this assumption is necessary or not. This section proposes an alternative, less restrictive sufficient condition.

The first sufficient condition consists in the fact that all trajectories of the process \( G(t, X(t, \omega)) \) reach a minimum value at the same instant \( t_{\min} \). It reads:

\[
\exists t_{\min} \in [0, T] \; \forall \omega \; \forall t \in [0, T] / \; G(t_{\min}, X(t_{\min}, \omega)) \leq G(t, X(t, \omega)) \tag{9}
\]

One important corresponding practical case can be identified. It corresponds to the following limit state function:

\[
G(t, X(t, \omega)) = a(\omega) - b(\omega).f(t) \tag{10}
\]

In terms of the classical \( R – S \) case, this expression corresponds to the following case:

- \( R(t, \omega) = R_\omega(t) - b(\omega).f(t) \) (of course restricted to \( R(t, \omega) > 0 \))
- \( S(t, \omega) \) is a constant but potentially uncertain load term or threshold, \( S(\omega) \)
- and consequently, \( a(\omega) = R_\omega(t) - S(\omega) \).

For the property to be valid, the sign of \( b(\omega) \) has to be constant (positive or negative random variable). For \( b(\omega) \) positive, \( t_{\min} \) corresponds to the instant where \( f \) reaches its maximum value on \([0, T] \). For \( b(\omega) \) negative, \( t_{\min} \) corresponds to the instant where \( f \) reaches its minimum value on \([0, T] \). Obviously, \( t_{\min} \) does not depend on \( \omega \). \( t_{\min} \) is bound to exist when \( f \) is assumed to be continuous.

\( b(\omega).f(t) \) reflects in fact the degradation of the structural resistance or material strength. It is assumed that the random part and the time dependence can be considered as independent.

It is well known that this expression is relevant for many degradation mechanisms, especially corrosion phenomena. In such cases, the variable \( b(\omega) \) may correspond to a constant or to a function of physical random input variables, \( b(\omega) = C(X_1, \ldots, X_n) \), and \( f(t) \) is often taken equal to \( t \), a power function of time. A complete review of degradation mechanism modeling is out of the scope of this chapter; however, it may be useful to give some illustrative examples. They are given in the sequel. Most of them are related to EDF power generation facilities. However, the degradation models mentioned in the following sections are illustrative and do not necessarily reflect the most recent models considered in EDF applications. They only show that a wide range of degradation models result in the standard decreasing limit state function case or in an extension of this case.

Two additional remarks regarding the relevance of this degradation model can be done by now.

First, it can be noticed that this extension could not apply to cumulated degradation mechanisms: when considering \( (b(\omega).f_1(t) + b_2(\omega).f_2(t)) \), the instant \( t_{\min} \) writes in fact \( t_{\min}(\omega) \) and depends on each trajectory.

Second, stochastic gamma processes are sometimes used to model the monotonic behavior of ageing and deteriorating processes (Mahmodian, 2013). However, this use should be examined very carefully. Indeed, the gamma process relies on the assumption of independent increases of the degradation level, which is not the case in the model suggested in (10). This assumption is not relevant as soon as the degradation kinetics is a monotonic function of certain physical input parameters. It comes that the gamma process cannot satisfactorily describe the physical behavior of most well-known degradation mechanisms. Its potential application should be limited to cases where the physical understanding of the phenomenon is not considered as satisfactory.

#### 3.1.1 Case of uniform corrosion affecting the metal

Models of corrosion rates are reminded in (Czarnecki, 2008) for steel bridges submitted to uniform corrosion due to various aggressive environmental conditions. This is one of the corrosion types that may affect a bridge: a popular classification based
on the visual aspect of corrosion effects has shown that eight corrosion forms may be active (Fontana, 1986). Uniform corrosion results in a loss of metal section of girders leading to a reduction of the load carrying capacity, and consequently a reduction of the bridge reliability. In addition, it may lead to a reduction of fracture and buckling resistances of a member. (Czarnecki 2008) considers that there is a common agreement that the corrosion loss versus time is best represented by the following exponential function:

\[ C = A \cdot e^{-Bt} \]  \hspace{1cm} \{11\}

where \( C \) is the corrosion loss after \( t \) years of exposure, \( A \) is the corrosion loss after one year of exposure, and \( B \) is a regression coefficient numerically equal to the slope of Eq. (12) in a log–log plot. The values of \( A \) and \( B \) depend on the type of steel and of environment. For instance, in case of carbon steel and marine environment (Czarnecki, 2008) states that \( A = 70.6 \mu \text{m} \) with the coefficient of variation equal to 0.66, and \( B = 0.789 \) with the coefficient of variation equal to 0.49.

It should be noted that in this model the degradation modeling does not correspond to the type "b(i(o), t(i))" suggested for extension in Equation 10, when \( B \) is taken random. However, if \( B \) is assumed to remain positive, each trajectory will be decreasing in time, which corresponds to the classical case mentioned at §2.2. This is another type of extension. However, this model does not account for any initiation time of corrosion, and finally (Czarnecki, 2008) suggests another modeling: three curves of penetration as a function of exposure time are given for high, medium and low corrosion rate; the corresponding analytical formulas are not given.

Similarly, (Ellingwood, 1995) considers that the depth \( \bar{X}(t) \) of deteriorated concrete or steel often can be modeled to an acceptable approximation by the following empirical model:

\[ \bar{X}(t) = C \cdot (t - t_i)^\alpha \]  \hspace{1cm} \{12\}

in which \( t \) is the time, \( t_i \) is the induction or initiation time to activate the deterioration process, \( C \) is a rate parameter and \( \alpha \) is a time-order parameter, assumed to be constant for a given corrosion mechanism. (Ellingwood, 1995) states that \( \alpha \) can be taken equal to \( \frac{1}{2} \) for diffusion-controlled processes (like corrosion of steel reinforcement), and may be greater than 1 for other mechanisms. \( C \) and \( t_i \) are considered as random variables. Again, this corresponds to the classical case mentioned at §2.2.

3.1.2 Case of corrosion of concrete reinforcement

Experience shows that for ageing concrete structures, the most important deterioration mechanism is reinforcement corrosion (Melchers 2008). The deterioration of the concrete is known to be comparatively small. One of the two mechanisms leading to reinforcement corrosion is concrete carbonation. It is a complex physico-chemical process that relies on the diffusion of CO\(_2\) into the gas phase of the concrete pores and its reaction with the calcium hydroxyl Ca(OH)\(_2\). The initiation of the reinforcement corrosion occurs when the carbonation front has reached the reinforcement, due to the progressive penetration of CO\(_2\) into the concrete mass by diffusion from the surface layer. The reinforcement bars in concrete structures are initially protected from corrosion by a microscopic oxide layer formed at their boundary due to the strong alkalinity of the pore solution. When the carbonation front reaches one bar, the surrounding pH strongly decreases (typically from 13 to 9), leading to corrosion initiation. The build-up of corrosion products eventually can lead to (longitudinal) cracking and spalling of concrete and loss of bond between the concrete and reinforcement. In other words, the volume of corrosion products causes tensile stresses that may be sufficiently large to cause internal micro-cracking and eventually spalling. These are destructive phenomena and are usually evident only after the occurrence of very significant reinforcement corrosion.

The durability of EDF cooling towers submitted to rebar corrosion induced by concrete (cement) carbonation has been investigated (Sudret 2005). As in another general reference (Melchers 2008), (Sudret 2005) assumes a constant corrosion rate \( k \) in the case of uniform corrosion and the corrosion model is considered to be linear after the initiation time:

\[
\begin{align*}
\text{d}(t) &= d_i \text{ if } t \leq t_{\text{in}} \\
\text{d}(t) &= \max(d_i - 2k_i \cdot t_{\text{corr}}(t - t_{\text{in}}), 0) \\
\text{d}(t) &= \max(d_i - 2k_i \cdot t_{\text{corr}}(t - t_{\text{in}}), 0)
\end{align*}
\]  \hspace{1cm} \{13\}

Where

- \( d_i \) : initial diameter of the rebar
- \( d(t) \) : diameter of the rebar at time \( t \)
- \( t_{\text{in}} \) : initiation time
- \( t_{\text{corr}} \) : corrosion current

The other mechanism responsible for reinforcement corrosion is the chloride ingress, that occurs in particular environmental conditions (presence of salt due to sea water or use of de-icing salt). Obviously, the cooling towers of the French nuclear power plants are not submitted to de-icing salts. Moreover, the plants that are built close to the sea do not use cooling towers but directly the sea water for cooling purposes. Thus, chloride induced corrosion cannot occur in this context (Sudret 2005).

3.1.3 Case of Flow-Accelerated corrosion

Flow-Accelerated Corrosion (FAC), also called corrosion-erosion, is a physico-chemical degradation mechanism affecting the carbon steel of the conventional part of nuclear power plants (Ardillon 1997). The surveillance of Flow Accelerated Corrosion (FAC) on secondary pipes has become a major concern for every nuclear power plant operator, especially after the Surry accident in 1986 (USA) and the Mihama accident in Japan (Persoz & al. 2006), both resulting in some deaths among the personnel of the power station that was present when the feedwater pipes burst out. The EDF monitoring strategy of FAC relies on the BRT-CIGERO\textsuperscript{TM} computer program.

Flow-accelerated corrosion is a physico-chemical process whereby the normally protective oxide layer on carbon or low-alloyed steel dissolves into a stream of flowing water or water-stream mixture. The oxide layer becomes thinner and less protective, and the corrosion rate is increased. Eventually a steady state is reached where the corrosion and dissolution rates are equal and stable corrosion rates are maintained. In some areas, the oxide layer may be so thin as to expose an apparently bare metal surface. More commonly, however, the corroded surface exhibits a black color typical of magnetite. FAC can be represented on Figure 1 below.
The four steps of the FAC mechanism described on Figure 1 are as follows:

1. Corrosion of carbon-steel by water and metal-oxide (magnetite) constitution
2. Soluble species diffusion: ferrous iron into oxide porosity & hydrogen into the metal
3. Oxide production at metal-oxide interface & oxide reduction at oxide-water interface
4. Diffusion of soluble species into bulk circulating water & hydrogen convection into air.

BRT-CICERO™ includes an algorithm to compute the FAC wear rate, considered as a kinetics evaluation. Therefore, the wall thickness loss is supposed to be linear in time and the performance function reads:

\[ G(t, X(t, \omega)) = E_{\text{res}}(t, \omega) - E_{\text{lim}} = E_{\text{ini}}(\omega) - E_{\text{lim}} - C(X_1, ..., X_n) \cdot t \]  

where:
- \( E_{\text{res}} \): residual thickness
- \( E_{\text{lim}} \): limit thickness ("failure" criterion)
- \( E_{\text{ini}} \): initial thickness
- \( X_1, ..., X_n \): physico-chemical inputs to the wear rate

More precisely, the thickness loss rate \( C(X_1, ..., X_n) \), denoted as \( V_{TL} \), is calculated as follows (Persoz & al. 2006):

\[ V_{TL} = \frac{\theta \cdot (C_{eq} - C_{\infty})}{K^* + (1 - f) \cdot (\frac{C_{eq}}{S} + \frac{C_{\infty}}{D})} \]  

where:
- \( \theta \): porosity of oxide layer,
- \( C_{eq} \): solubility of ferrous ion in water in equilibrium with magnetite reduction,
- \( C_{\infty} \): ferrous ion concentration in bulk water,
- \( K^* \): kinetic coefficient of formation of ferrous hydroxyde \((Fe + 2H_2O \rightarrow Fe(OH)_2 + H_2)\),
- \( f \): rate of ferrous ion soluble and transformed in magnetite at Metal-Oxide interface (\( f \) is mainly equal to 0.5),
- \( k \): mass transfer coefficient,
- \( \delta \): oxide thickness,
- \( D \): diffusion coefficient of ferrous ion in water.

\( G(t, X(t, \omega)) \) is clearly a decreasing function in time (cf. section 2.2).

Note that the same type of expression of the \( G \) function as denoted in Equation 14 is valid for cavitation (erosion by cavitation), another significant degradation mechanism affecting PWRs for which the thickness loss kinetics is linear in time and relies on a wear rate evaluation.
3.1.4 Conclusion
This short review confirms that the case of decreasing performance function $G$ is of significant practical importance in industrial time-variant structural reliability analyses. All the mechanisms investigated in this section, having an importance for the lifetime of EDF nuclear facilities, are related. A possible extension of this case is given, valid when it is possible to separate, in the $G$-function, a temporal part and a random part.

3.2 How to integrate some realistic physical considerations

It is reminded in this part that for some physical problems (at least for ultimate limit states) multiple outcrossings are impossible: structures cannot fail twice. In other words, the occurrence of a failure means that no previous failure has occurred. Obviously, this point of view considers the structure as a unique physical device: failure means structural loss. In this regard, maintenance operations or replacements are considered to be excluded or to lead to a different structure. For instance, this is the case of non-repairable components. Moreover, this assumption reflects the situation of ultimate limit states (e.g. brittle fracture, plastic collapse) rather than serviceability limit states, for which an outcrossing only means that the structural operation can continue, although in a degraded condition. By extension, it is relevant for any limit state considered as ultimate (for safety reasons), although its definition includes an implicit safety margin (Ardillon, 2009). This extension is relevant for risk-sensitive industries: in that case, the structural end of life may result not only from a physical loss, but also from a regulatory breakdown initiated by the violation of a safety criterion. This assumption has been considered in global risk evaluations of nuclear components (Ardillon, 2010). The impact of this consideration is investigated and the fundamentals of an adequate treatment are proposed in this section. Further developments are given in section 3.3.3.

The way to integrate this context is to consider, for one single state on a given time interval $[0, T]$, the scalar process representing the $G$ function until the first outcrossing, and then to consider $G$ equal to zero. This can be done by considering a new process denoted by $\hat{G}(t, X(t, \omega))$, which will represent the physical reality: the part of the anticipated trajectory after the instant to failure does not exist in fact, since it corresponds to the structural end of life due to physical loss or regulatory reasons. Let us define:

$T_G(\omega) = \inf\{u \in [0, T] / G(u, X(u, \omega)) = 0\}$. $T_G(\omega)$ is the time of first outcrossing. The appropriate process is now:

$G(t, X(t, \omega)) = G(t, X(t, \omega)) \cdot 1_{\{T_G(\omega) < t\}}$ \hspace{1cm} (16)

The following figure shows the two processes.

![Figure 3. A common trajectory of the $G$ and $\hat{G}$ processes.](image)

The trajectories of these two processes are not necessarily decreasing. The following relationships can be easily shown:

$T_{\hat{G}}(\omega) = T_G(\omega)$ \hspace{1cm} (17)

$P_{\hat{G}}(0, t) = P_G(0, t)$

Moreover, for the process $\hat{G}$, instantaneous and global failure probabilities are identical:

$P_{\hat{G}}(t) = P_{\hat{G}}(0, t)$ \hspace{1cm} (18)

This property is interesting, since it means that in the general case where only the assumption of unique failure is considered, it is possible to generalize the case of the decreasing margin for the theoretical evaluation of the global failure probability.

For practical application, it should be seen that the exploitation of this property is rather intended for Monte Carlo simulation methods, since it is necessary to simulate trajectories, which can be time consuming for mechanical problems, unless surrogate models can be used. However, it makes it possible to stop the simulation at the failure instant, thus sparing some computational time. Moreover, trajectory simulations are popular in some areas, like non-linear structural dynamics; the definition of the $G$-
function through a stochastic differential equation may be a favorable issue, since it could give a natural discretization scheme, as in financial mathematics (e.g. Black & Scholes or Ornstein-Uhlenbeck processes). The usefulness of this property for non-simulation approaches should be investigated.

3.3 Outcrossing rate and classical failure rates

Reliability evaluations are performed for many types of industrial components, not only for structures. For safety reasons, industrial structures that constitute the NPPs facilities experience very few failures; when no failure experience feedback is available, the only method that can be used to evaluate their reliability is structural reliability. In the contrary, active components like pumps or valves may experience more failures. In that case, a reliability evaluation can be made based on statistical analysis of experience feedback, possibly including expert judgement. This will be called in this chapter classical reliability.

These two situations have led to two different types of methods, and also to two different kinds of indicators. Whereas time-variant structural reliability theory relies on the two main indicators shown in §2 (namely the global failure probability and the outcrossing rate \( \nu^+(t) \)), classical reliability basically provides failure rates. Two main failure rates are available:

- The usual failure rate \( \lambda(t) \), also called hazard function or conditional failure rate (Melchers, 1999), defined as follows at a given instant t:

\[
\lambda(t) = \lim_{\Delta t \to 0} \frac{P(T \leq T + \Delta t | T > t)}{\Delta t}
\]

where \( P \) stands for “probability” and \( T \) denotes the 1st failure time (random variable).

- The Vesely failure rate \( \lambda_V(t) \), defined as follows at a given instant t (Cocozza-Thivent, 1997):

\[
\lambda_V(t) = \lim_{\Delta t \to 0} \frac{P(S_{t+\Delta t} = "Breakdown" | S_t = "Running")}{\Delta t}
\]

where \( P \) stands for “probability” and \( S_t \) denotes the component state at time t.

The difference between the two rates is that \( \lambda_V(t) \) is used to integrate maintenance effects, especially corrective maintenance: various failures may have occurred before time t, but the corresponding maintenance actions have succeeded.

3.3.1 General relationship

From Equation 25 it comes:

\[
\lambda_V(t) = \lim_{\Delta t \to 0^+} \frac{P(G(t + \Delta t, X(t + \Delta t, \omega)) \leq 0 | G(t, X(t, \omega)) > 0)}{\Delta t}
\]

Let us denote:

\[
\frac{P(G(t + dt, X(t + dt, \omega)) \leq 0 \cap G(t, X(t, \omega)) > 0)}{dt}
\]

Then it comes:

\[
\lambda_V(t) = \lim_{\Delta t \to 0^+} \frac{r_1(dt)}{1 - P(G(t, X(t, \omega)) \leq 0)}
\]

From Equations 2 and 5 one gets:

\[
\lambda_V(t) = \frac{\nu^+(t)}{1 - \nu^-(t)}
\]

It is interesting to point out that a very simple relationship exists between \( \nu^+(t) \) and \( \lambda_V(t) \). For low instantaneous failure probabilities, the two quantities are almost equal.

Similarly, it can be seen from Equation 24 that:
Second, since the conditioning events are equivalent in this case.

According to the definition of the global failure probability (cf. Equation 1), one gets:

\[ \dot{\lambda}(t) \leq \frac{\nu^*(t)}{1 - P_f(t)} \]  \hspace{1cm} (26)

where \( P_f(0,t) \) denotes the global failure probability for the time interval \([0,t]\).

### 3.3.2 Case of decreasing performance function

Let us consider now the case of decreasing performance function \( G \). Considering the definitions given in Equations 24 and 25, it can be seen that in this case \( \lambda(t) \) and \( \lambda^*(t) \) are identical. Consequently, \( \dot{\lambda}(t) = \lambda^*(t) = \frac{\nu^*(t)}{1 - P_f(t)} \) \hspace{1cm} (27)

### 3.3.3 Case of unique failure

The fundamentals of this case have been given in section 3.2. The process to be considered is therefore \( \dot{G}(t, X(t, \omega)) \), which best represents the physical reality. The developments presented in this section are related to failure rates and outcrossing rate. First, it comes from Equations 24 and 25 of the failure rates that in this case \( \lambda(t) \) and \( \lambda^*(t) \) are identical (for the process \( \dot{G}(t, X(t, \omega)) \), since the conditioning events are equivalent in this case.

Second, since \( T_L(\omega) = T_L(\omega) \), \( \dot{\lambda}^*(t) = \dot{\lambda}^*(t) \).

Combining these two results with Equation 29, it comes finally:

\[ \dot{\lambda}^*(t) = \dot{\lambda}^*(t) = \frac{\nu^*(t)}{1 - P_f(t)} \]  \hspace{1cm} (28)

or, equivalently:

\[ \dot{\lambda}^*(t) = \dot{\lambda}^*(t) = \frac{\nu^*(t)}{1 - P_f(0,t)} \]  \hspace{1cm} (29)

Again, it is possible to generalize the case of the decreasing margin for the relationships between failure rates and outcrossing rate.

These investigations show the closeness of the two classes of reliability methods, often considered as different knowledge areas. They also give a precise sense to the usual failure rates in the framework of time-variant structural reliability theory.

### 4 CONCLUSION

This chapter has presented some theoretical enhancements to the background of time-variant structural reliability theory. It is well known that the case of a decreasing G-function (margin) is of great practical importance: in this case the global failure probability reduces to an instantaneous probability.

A short review of degradation mechanisms has been performed and has confirmed that the case of decreasing performance function \( G \) is of significant practical importance in industrial time-variant structural reliability analyses. All the mechanisms investigated in this study, having an importance for the lifetime of EDF nuclear facilities, are related.

Then, three possible extensions have been proposed. First, for a general process the FORM evaluation needs only the evaluation of the maximum instantaneous probability. Second, a more general form of the G-function leads to the same situation: it may appear when it is possible to separate a temporal part and a random part. The third extension is the case of "unique failure", for which it is considered that a structure cannot fail twice (e.g. case of non repairable components). In this case the global failure probability also reduces to the probability at the end of the time interval.

Finally, this chapter has investigated the relationship between the outcrossing rate \( \nu^*(t) \) (key concept of the time-variant structural reliability theory) and failure rates commonly used in the classical reliability theory. Thus, simple relationships are easily derived between \( \nu^*(t) \) and the usual failure rate \( \lambda(t) \) and the Vesely failure rate \( \lambda^*(t) \). They have been adapted to the particular cases considered in this work (decreasing margin, unique failure). These investigations show the closeness between the two classes of reliability methods, often considered as different knowledge areas. They also give a precise sense to the usual failure rates in the framework of time-variant structural reliability theory.

These new theoretical enhancements can contribute to facilitate the application of time-variant reliability theory.
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